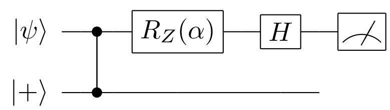
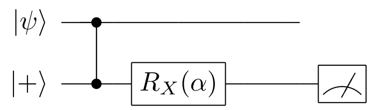
One-way Quantum Computation - Exercises

1. Consider the following two quantum circuits acting on a pure single qubit input state and an ancilliary state prepared in $|+\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$. The first is:



(1)

and the second is:

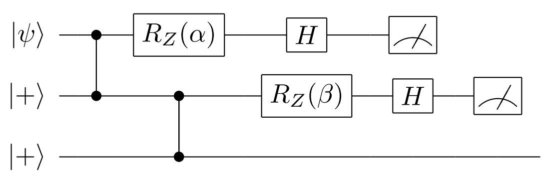


(2)

All measurements here are in the computational basis.

Given the input state $|\psi\rangle = \alpha\,|0\rangle + \alpha\,|1\rangle$, calculate the output state of these two circuits. How does the outcome of the measurement effect the output state? What is the probability of the different measurement outcomes in each case, and how much information does the measurement reveal about the input state? What simple element could you add to these circuit to make the output state independent of the measurement outcome? (Hint: you are allowed to add a circuit element which depends upon the measurement outcome).

2. Consider the following quantum circuit, constructed by concatenating two copies of circuit 1:



(3)

Without explicitly working through the circuit gate by gate, calculate the output state of this circuit. Let $m = \{0, 1\}$ be the outcome of the measurement of the uppermost qubit. Set the angle of rotation on the second qubit β to $(-1)^m\theta$. Is it now possible to make the output state completely independent of the measurement angles by the addition of an extra simple circuit element, as above? Would this be possible if all measurements were made simultaneously?

Explain why there would be little point in directly concatenating circuits of the form of circuit (2) above.

The following definitions may be helpful:

$$R_Z(\alpha)\,|0\rangle = e^{-i\frac{\alpha}{2}}\,|0\rangle$$
$$R_Z(\alpha)\,|1\rangle = e^{i\frac{\alpha}{2}}\,|1\rangle$$
$$R_X(\alpha)\,|0\rangle = \cos(\alpha/2)\,|0\rangle - i\sin(\alpha/2)\,|1\rangle$$
$$R_X(\alpha)\,|1\rangle = -i\sin(\alpha/2)\,|0\rangle + \cos(\alpha/2)\,|1\rangle$$

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